Some Supersymmetric Flavour Problems

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1 Preliminary Remarks and Summary

In the phenomenological analyses of supersymmetric extensions of the Standard Model, one has to face a plethora of new parameters. Most of them are related to the masses of the sfermions – squarks and sleptons. Of course this is also true in the Standard Model because of the flavour problem: three families of fermions with hierarchical masses and mixings. It becomes much worse in the supersymmetric extensions of the Standard Model, since for each fermion one has two scalars. Only the mass eigenvalues and the relative mixings plus one phase, in the left-handed sector are observable in the Standard Model. The situation is more involved in the susy extensions, where there are twice more eigenvalues and many mixings and phases become observable.

There are many experimental – and phenomenological – constraints on these parameters, but they are somewhat difficult to analyse without more or less arbitrary assumptions. Still they provide very strong limits on many of these parameters. Basically, these bounds come from the fact that the new particles could excessively contribute to processes that are in good agrement with the Standard Model or for which there exist experimental limits. How to exploit this information to understand the origin of this parameters: the supersymmetry breaking mechanism?

The fermion masses and mixings have motivated many suggestions on the possible origin of the flavour structure in the Standard Model. Most of them introduce new – flavour – symmetries. What can these models tell us about the masses and mixings of sfermions? Are their predictions consitent with the experimental limits?

These lecture notes present a necessarily limited appraisal of these questions. Since several other lectures were presented in the same School, some problems are to be found in the other contributions to these proceedings. These notes concentrate on the more qualitative side of the various questions related to the flavour patterns for the squark and slepton masses. They try to remain simple more than complete. The precise phenomenological analysis and the many models for the sfermion masses are available in the references herein and they are not exhaustively listed below.

Since this is not a review, just lecture notes, the references are only to papers that were mostly useful in preparing them, hence very subjective. I apologize to all the authors that were forgotten or not systematically copy-pasted to the included short bibliography.

These notes are organized as follows. We first discuss the general structure of the parameters related to the sfermion masses, in particular,

- General form of the mass matrices
- Universality of soft terms

Next we recall the main phenomenological constraints on these parameters, skipping the important chapter of CP violations discused by Y. Nir [1]. The main subjects are then,

- Baryon and lepton number and R-parity
- Charge and colour breaking
- Supersymmetric FCNC problems
- Solutions: degeneracy, alignment and decoupling

In order to understand the relation between the soft parameters and the breaking of supersymmetry, the next section summarizes the relevant aspects of broken supergravity. The basics of sugra and much more are contained in the R. Grimm lecture notes [2], we just concentrate on a few points,

- Kähler geometry
- Supersymmetry breaking
- Origin of soft terms
- Soft terms as remnants of hidden susy breaking

The emphasis is on the geometrical aspect of the transmission of supersymmetry breaking and the influence of new physics into the Kähler potential. The content and the use of the resulting formulae is applied to a few examples:

- Modular invariant Kähler geometry
- Gauge mediated supersymmetry breaking
- Decoupling of a flavour theory

Finally, some recent ideas related to these problems are illustrated by developping an example that is not quite realistic but still presents many of the features that arerelevant in the subject. This is preceeded by an introduction of some of the ingredients,

- Froggatt-Nielsen paradigm
- Green-Schwarz mechanism
- Model with anomalous U(1)

The aim of this discussion is to put forward the impact of a flavour theory on the pattern of scalar sparticles, and, hopefully in the future, the impact of the pattern of scalar sparticles on a flavour theory .

Many other aspects were discussed in the other lectures and were not included here, in particular, problems related to grandunification (R. Barbieri)[3] and string phenomenology (G. Ross)[4], the most appealing frameworks to face the flavour problem after all.

2 Flavour Patterns of Scalar Masses

2.1 General form of the mass matrices

Our first task is to identify the possible patterns of the susy breaking parameters. In the extensions of the Standard Model with the minimum field content and R-parity, or MSSM, these are the only new parameters since the gauge and Yukawa couplings are known, and they are all related to the masses of the susy particles: gaugino masses and scalar masses. Therefore they control essentially all the expected effects that could supply any evidence for susy in Nature. In these notes we shall concentrate on the scalar masses and mention gaugino masses only when they become relevant in the discussion.

The plethora of new parameters in the scalar mass sector is mostly due to our ignorance about the origin of flavour. One is forced to allow for the most general structure, only constrained by the experimental and phenomenological limits. Besides the two doublets of Higgs scalars denoted H_1 and H_2, the MSSM scalar matter has 15 matter complex scalars, one for each fermion in the SM, which we denote:

$$Q_L^i = \left(\begin{array}{c} U_L^i \\ D_L^i \end{array} \right) \quad U_L^{Ci} = U_R^{i*} \quad D_L^{Ci} = D_R^{i*} \quad L_L^i = \left(\begin{array}{c} N_L^i \\ E_L^i \end{array} \right) \quad E_L^{Ci} = E_R^{i*} \tag{1}$$

where $i = 1, 2, 3$, is a family index. The scalars associated to the left-handed quarks and leptons are grouped in doublets by the electroweak symmetry.

Consider the MSSM superpotential,

$$W_{MSSM} = h_{ij}^U Q^i H_2 U^{cj} + h_{ij}^D Q^i H_1 D^{cj} + h_{ij}^E L^i H_1 E^{cj} + \mu H_1 H_2, \tag{2}$$

The resulting (globally) supersymmetric scalar potential is

$$V_{susy} = \sum \left| \frac{\partial W}{\partial z^i} \right|^2 + \frac{1}{2} \sum (D^\alpha)^2 \tag{3}$$

where $z^i = H_1, H_2, U, D, Q, L, E$, $D^\alpha = \bar{z}^{\bar{i}}(T^\alpha z)^i$, and T^α are the corresponding representation of $SU(3) \otimes SU(2) \otimes U(1)$. The soft part, *i.e.*, the susy breaking terms of the scalar potential are (flavour indices omitted):

$$\begin{aligned} V_{soft} &= \sum m_i^2 |z^i|^2 + A_U h^U Q H_2 U^c + A_D h^D Q H_1 D^c \\ &+ A_E h^E L H_1 E^c + B\mu H_1 H_2 \end{aligned} \tag{4}$$

The general mass matrices for these scalars (assuming R-parity) are of the form:

$$\begin{pmatrix} U_L^\dagger & U_R^\dagger \end{pmatrix} \begin{pmatrix} \tilde{m}^2_{U_L} & \tilde{m}^2_{LR} \\ \tilde{m}^{2\dagger}_{LR} & \tilde{m}^2_{U_R} \end{pmatrix} \begin{pmatrix} U_L \\ U_R \end{pmatrix} \tag{5}$$

where $\tilde{m}^2_{U_L}, \tilde{m}^2_{U_R}, \tilde{m}^2_{LR}$ are three by three matrices in family space. There are analogous mass matrices for the D-squarks, the E-sleptons and the sneutrinos N. Taking into account the electroweak symmetry breaking by the Higgs *vev*'s, which give masses to the quarks and leptons, one obtains the usual expressions:

$$\begin{aligned} \tilde{m}^2_{U_L} &= \tilde{m}^2_Q + m_u^\dagger m_u & \tilde{m}^2_{U_R} &= \tilde{m}^2_U + m_u m_u^\dagger \\ \tilde{m}^2_{D_L} &= \tilde{m}^2_Q + m_d^\dagger m_d & \tilde{m}^2_{D_R} &= \tilde{m}^2_D + m_d m_d^\dagger \\ \tilde{m}^2_{E_L} &= \tilde{m}^2_L + m_e^\dagger m_e & \tilde{m}^2_{E_R} &= \tilde{m}^2_E + m_e m_e^\dagger \\ & & \tilde{m}^2_{N_L} &= \tilde{m}^2_L + m_\nu^\dagger m_\nu \end{aligned} \tag{6}$$

where m_u, m_d, m_e and m_ν are the fermion mass matrices, in a general basis in the family space.

It is usual and useful to define the matrix elements that connect L and R scalars in (5 in terms of the A_U, A_D, A_E matrices (in flavour space) by:

$$\begin{aligned} \tilde{m}^2_{U,LR} &= (A_U + \mu \cot\beta)\, m_U \\ \tilde{m}^2_{D,LR} &= (A_D + \mu \tan\beta)\, m_D \\ \tilde{m}^2_{E,LR} &= (A_D + \mu \tan\beta)\, m_E \end{aligned} \tag{7}$$

which separates the contributions (A matrices) from susy breaking and the supersymmetric term proportional to the μ parameter, with $\tan\beta = \langle H_2 \rangle / \langle H_1 \rangle$. This defintiton of the A matrices is general but not necessarily the most natural as it will become apparent from the their relation to susy breaking discussed below.

Let us now count the number of parameters in these soft terms. There are five hermitian matrices $\tilde{m}^2_\Phi$, $\Phi = Q,\ U,\ D,\ L,\ E,$, and 3 complex matrices, A_U, A_D, A_E, hence 99 parameters altogether (100 if one counts the supersymmetric Higgs mass μ)! In the absence of further constraints, they are all of the order of the susy breaking scale as felt by the matter scalars, denoted by m_{susy}. Notice that the A parameters appear multiplied by the fermion masses, so that their physical relevance is diminished, but for those multiplying masses of the third family and, specially, m_{top}.

In order to define these parameters in an unambiguous way, we choose to work in the basis of the family space where m_u and m_e are diagonal, unless otherwise specified. This is the basis of the physical up-quarks and charged leptons, which turns out to be convenient. The left-handed down-quarks differ from the physical states by a CKM transformation. This gives a definition of 99 parameters unconstrained by the SM symmetries, that we now turn to analyse.

Of course, there would be a tremendous simplification of the flavour dependence if one could find a reason to impose that the sfermion mass matrices (5,6,7) and the corresponding fermion mass matrices are diagonal in the same basis. One of the main scopes of these lectures will be to emphasize that this is a difficult task since the origins of the two kinds of mass matrices are completely different: one is due to susy breaking and the other one is due to the electroweak symmetry breaking.

Up to now we have diagonalized the fermion masses, or the Yukawa couplings, by unitary transformations of the chiral multiplets and so defined the CKM matrix. If now we define the physical squarks by the eigenvectors of (5), the supersymmetry transformations on physical states will connect states with different flavours. E.g., for the physical down-quarks and the corresponding squarks, one would have for the action of the susy charge $Q_{1/2}$:

$$Q_{1/2}\begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} *** \\ *** \\ *** \end{pmatrix}\begin{pmatrix} \tilde{d} \\ \tilde{s} \\ \tilde{b} \end{pmatrix} \quad \Longrightarrow \text{flavoured susy}$$

Susy is not flavour diagonal anymore and relates physical states from different families.

This has an important consequence for the couplings of the susy particles, which in a supersymmetric theory are related to the Yukawa and gauge couplings of the SM. For instance, conservation of colour and e.m. symmetries ensure that the gluon and photon couplings in the lagrangian are flavour diagonal in the physical states. Supersymmetry transmits this property to the gluino and photino couplings. After susy breaking, the action of supersymmetry is no more flavour diagonal, so that the gluino and photino will couple fermions from sfermion of different families and produce flavour mixing. These effects will be proportional to the misalignment between fermion and sfermion masses, namely, to the commutators:

$$\begin{gathered}\left[\tilde{m}_Q^2, m_u^\dagger m_u\right] \neq 0 \qquad \left[\tilde{m}_Q^2, m_d^\dagger m_d\right] \neq 0 \qquad \left[\tilde{m}_U^2, m_u m_u^\dagger\right] \neq 0 \\ \left[\tilde{m}_D^2, m_d m_d^\dagger\right] \neq 0 \qquad \left[\tilde{m}_L^2, m_e^\dagger m_e\right] \neq 0 \qquad \left[\tilde{m}_E^2, m_e m_e^\dagger\right] \neq 0 \\ \left[A_U^\dagger A_U, m_u m_u^\dagger\right] \neq 0 \quad \left[A_D^\dagger A_D, m_u m_u^\dagger\right] \neq 0 \quad \left[A_E^\dagger A_E, m_u m_u^\dagger\right] \neq 0 \end{gathered} \tag{8}$$

Actually, radiative corrections to the scalar matrices are important in the MSSM and are nicely related to the electroweak symmetry breaking [5] : the large Yukawa coupling of the t quarks to H_2 Higgs is instrumental, together with the fact that the H_2 (mass)^2 is more renormalized by the radiative corrections than the others. It is then naturally negative at a scale that has to be close to 174GeV, where $\langle H_2 \langle \neq 0$, while the stop (mass)^2 are still positive to prevent colour and charge symmetry breaking. The radiative corrections due to the highly hierarchical Yukawa couplings will introduce further flavour dependence on the sfermion mass matrices. Instead, radiative corrections due to gauge couplings are family independant, adding a term proportional to the unity matrix to (5) and reducing the family dependence.

A final remark on the important issue of CP violation. The phases of the matter supermultiplets have been redefined so that the fermion masses are real. The phases of the complex sfermions are therefore fixed or, if the parameters in (6, 7) are made real, the phases will be retrieved in the couplings of the sfermions, just like for fermion mixing. These are therefore new sources of CP violation which have to be carefully confronted to experiments [1].

2.2 Universality of soft terms

Of course, the number of soft masses can be drastically reduced if one assumes [6] that all the parameters of the same nature, $\tilde{m}^2$ or A, are identical,

$$\begin{aligned} \tilde{m}_\Phi^2 &= m_0^2 \mathbf{1} \quad , (\forall \Phi) \\ A_\Phi &= A_0 \mathbf{1} \quad , (\forall \Phi) \end{aligned} \tag{9}$$

Mostly often, this assumptions are accompanied by that of gaugino universality, namely, that the gaugino masses are all equal,

$$m_{\tilde{g}} = m_{\tilde{W}} = m_{\tilde{B}} = M_{1/2} \tag{10}$$

Of course this universality is not, by far, a symmetry of the supersymmetric theory. It is badly broken by both the gauge interactions – since fermions in each family have different gauge quantum numbers – and the Yukawa couplings which are hierarchical and carry the information on family mixing. Therefore universality will receive quantum corrections. If the conditions (9) are imposed at some appropriate high scale, then they will be violated by radiative corrections whose main effect will be the running down to the low scale where susy is broken and the SM obtained.

As already stressed the gauge interactions just add a different term proportional to the unity matrix to each one of the $\tilde{m}_\Phi^2$ and A^Φ, proportional to the gaugino (mass)^2, hence to $M_{1/2}^2$, and they do not affect the family independence. The radiative corrections from Yukawa couplings introduce a family dependence instead. For instance, if one assumes (9,10) at a high scale Λ (the grandunification sclae or $M_{\rm Planck}$ being natural choices) and solve the one-loop RGE [7] in the case $Y_t \gg Y_b$ (hence $Y_c \gg Y_\lambda$ one obtains at hte scale of $O(M_Z)$

(the generated non-diagonal entries are negligible in the basis above-defined),

$$\begin{aligned}
\left(\tilde{m}_Q^2\right)_{11} - \left(\tilde{m}_Q^2\right)_{33} &\approx (Y_t/Y_{\rm crit})^2 \left(\frac{1}{2}m_0^2 + M_{1/2}^2\right) \\
\left(\tilde{m}_Q^2\right)_{11} - \left(\tilde{m}_Q^2\right)_{22} &\approx (Y_c/Y_{\rm crit})^2 \left(\frac{1}{2}m_0^2 + M_{1/2}^2\right) \qquad (11)
\end{aligned}$$

where Λ has been taken at the grandunification scale, and where $Y_{\rm crit}$ is the value of $Y_t(m_t)$ such that $Y_t \to \infty$ at the scale Λ (Landau pole). For $\Lambda \approx M_{\rm Planck}$, $Y_{\rm crit} \approx 1.1$. The coefficients have been roughly approximated. In order to estimate the effect of this renormalization on the misalignement defined by (8) with respect to m_d, the down-quark mass matrix, let us multiply (11) by the appropriate matrix element of the CKM matrix:

$$\begin{aligned}
&\frac{\left(\tilde{m}_Q^2\right)_{11} - \left(\tilde{m}_Q^2\right)_{22}}{\left(\tilde{m}_Q^2\right)_{11} + \left(\tilde{m}_Q^2\right)_{22}} \times \theta_{12} \approx 3.10^{-6} \times \frac{m_0^2 + 2M_{1/2}^2}{m_0^2 + 7M_{1/2}} \\
&\frac{\left(\tilde{m}_Q^2\right)_{11} - \left(\tilde{m}_Q^2\right)_{33}}{\left(\tilde{m}_Q^2\right)_{11} + \left(\tilde{m}_Q^2\right)_{33}} \times \theta_{13} \approx 2.10^{-3} \times \frac{m_0^2 + 2M_{1/2}^2}{m_0^2 + 7M_{1/2}^2} \qquad (12)
\end{aligned}$$

If the scale Λ is taken at a much lower scale, as in gauge mediated models, the reduction of these quantities is roughly proportional to $Y_{\rm crit}^{-2}$, which becomes larger for low Λ. Therefore the effects of family dependence remain small or moderate even if universality is assumed at large Λ. In this sense the universality assumption is quite consistent with the approximate conservation of flavour in $\Delta Q = 0$ transitions. Of course one has to relate this property to the susy breaking mechanism. This will be discussed below.

3 Phenomenological Constraints

There are several phenomenological problems that are related to the flavour structure – including the related CP violation due to phases in the mass parameters. Many of them are related to rare processes that could be dangerously enhanced from the radiative corrections due to sparticles in quantum loops. In order to comply with all the constraints so obtained one has to restrict the allowed range for the parameters, specially for the soft terms. Let us briefly recall the main questions to be investigated.

3.1 Baryon and Lepton numbers

Problems:
In the SM, the conservation of B and L is said to be "accidental" as far as it is a consequence of the gauge symmetries and of the field content of the SM and its renormalizability. The introduction of the matter scalars allow for

more interactions that violate B and L and are invariant under the SM gauge symmetries. Basically, this can be understood from two facts:

(i) One of the Higgs doublets (H_1) that couples to down-quarks and to leptons, has the same quantum numbers as the three supermultiplets L^i that contain the left-handed lepton doublets except for L. Therefore H_1 and L^i are alowed to have exactly the same interactions and the following gauge invariant terms :

$$L^i L^j E^k, \quad L^i D^j Q^k, \quad \mu_i L^i H_2 \implies \mathrm{L-vioitg}, \tag{13}$$

all vioitg L, are allowed in the superpotential.

(ii) The antisymmetric cubic QCD invariant allows for some Yukawa couplings of two quarks to a squark, which corresponds to terms of the form

$$U^k D^i D^j \qquad \implies \mathrm{B-vioitg} \tag{14}$$

in the superpotential, which violates B. Notice that in the presence of these couplings the scalar behave as the so-called leptoquarks, leading to the same kind of exotic physics. These are the so-called R-vioitg couplings.

There is a very strong limits on some of those couplings if they are all present in some combinations. The most important one, comes from proton decay. It goes without saying that the resulting bounds are very small numbers. Of course this can be avoided by imposing B, so relaxing the limits on the L vioitg couplings. But the latter mix the neutral higgsino, the H_1^0 partner, to the neutrinos, giving a mass to one of them at a level which is already cosmologically forbidden. There are also many other limits from rare and exotic processes and precision tests [8]. Some couplings are less restricted than the others.

There are two consequences if the couplings (13) and/or (14) are allowed at the levels allowed by the phenomenological limits, since there is no stable supersymmetric particle (LSP) anymore: there would be no natural susy candidate for the dark matter and susy events at colliders would miss their characteristic missing energy signature.

Possible Solutions:

(A) There have been attempts to solve this problem by extending flavour models for the quark and lepton masses to derive a pattern for the couplings (13) and (14) consistent with the phenomenological limits, including the neutrino masses and proton decay [9]. In spite of this existence proof, these models are not very appealling. Also, we all like the susy dark matter candidate, don't we?

(B) There is a very elegant solution to banish all the B and L vioitg couplings (13) and (14) from the superpotential by assuming a discrete symmetry, the R-parity, which is +1 for the SM particles and -1 for their susy partners. In this way one recovers the B and L conservation (in the effective renormalizable low energy theory at least). This is the option chosen in the MSSM and we shall assume the R-parity from here on.

3.2 Charge and Colour Breaking

Problems:

Vacuum stability bounds are a particularly important issue for supersymmetric models because of the large number of scalars, any of which can get a vacuum expectation value, possibly breaking charge and/or colour. Insisting on the physical vacuum being stable results in a set of constraints on the possible supersymmetry breaking parameters which are generally known as Charge and Colour Breaking (CCB) bounds [10]. There are different schools of thought regarding the precise cosmological meaning of these bounds. As the vacuum choice depends on unknown details of our cosmological history, *e.g.*, the reheating temperature, CCB minima should ultimately be regarded as a constraint on early cosmology rather than particle physics. Henceforth we take the view that areas of parameter space which have CCB minima are simply less likely because their cosmology is severely restricted.

Consider the MSSM potential in (2, 3, 4). Notice that V_{susy} is quartic in the scalar fields, while V_{soft} is trilinear. Then, V_{susy} dominates for large values of the fields, $z^i \gg m_{susy}$, the characteristic scale of the m_i and A in (4).

The CCB minima approximately occur along directions in the scalar field space where the terms in the scalar potential with gauge couplings vanish, namely, where $\langle D^\alpha \rangle = 0, \forall \alpha$ *(D-flat directions)*. These directions are in a one-to-one correspondence with the solutions of the equation [11]

$$\frac{\partial I(z)}{\partial z^i} = C \bar{z}^i \tag{15}$$

where $I(z)$ is any analytic polynomial invariant under the gauge group, and C is a complex number (when the metrics take a general Kähler form to be discussed, $\bar{z}^i$ is replaced by K_i. Since $I(z)$ is an analytic invariant, $\langle D^\alpha \rangle = 0$ at any solution of (15).

The MSSM superpotential (2) is a gauge invariant, so that we can choose $I = QH_2U^c$, which satisfies (15) along the direction $Q = H_2 = U^c$(indices omitted). This is the D-flat direction. It is easily checked that, if $m_Q \approx m_U \approx m_{H_2}$ and $h_U \ll 1$, the scalar potential, $V_{susy} + V_{soft}$, has a minmum close to this direction, with $Q \approx A_U/h_U$ if $m_Q \approx m_U \approx m_{H_2}$ and $h_U \ll 1$(this excludes the third family), *unless:*

$$|A_U|^2 < 3(m_Q^2 + m_U^2 + m_{H_2}^2) \tag{16}$$

where the parameters are to be calculated at a scale $\lambda \sim Q \approx A_U/h_U$. which is relatively large for $h_U \ll 1$. Since $m_{H_2}^2$ is decreasing with the scale, to produce the radiative breaking of the electroweak symmetry, (16) could be violated at this scale even when it is not at higher ones.

Therefore one obtains a condition analogous to (16) for the A parameters associated to each of the cubic terms in (4). This yields a reduced allowed region in the parameter space of the MSSM, hence a constraint on the susy breaking mechanism. The actual condition has to be investigated numerically since, as already mentioned, (16) is only approximate.

However, the most severe conditions come from directions that are both

D-flat and F-flat,, namely that satisfy (15) for some invariant $I(z)$ *and:*

$$\frac{\partial W}{\partial z^i} = 0 \tag{17}$$

Along these directions $V_{susy} = 0$. They have been called UFB (acronym of a meaningless denomination which is needless recalling). The invariant in (15) is better taken different from those in (2), so that (17) is possible. Actually, the UFB are associated to gauge invariants that are excluded from the superpotential, a privilege which is granted by susy. This is precisely the case for the invariants (13) and (14) which have been forbidden by our assumed R-parity.

Consider the invariant,

$$I = L(QD + \mu' H_2) \tag{18}$$

Then, imposing (15) and (17) one fixes μ' and obtains a direction which is *D-flat and F-flat,*

$$\begin{aligned} H_2 &= -a^2\mu/h_D \\ Q &= D = a\mu/h_D \\ L &= a\sqrt{1+a^2}\mu/h_D \end{aligned} \tag{19}$$

Along this direction, the potential is controlled by the (mass)^2 terms in V_{soft} and could develop a relevant local minimum at some scale due to the running of the parameters. The absence of this minimum requires some inequalities for the soft parameters in the running (mass)^2. For instance, in the MSSM with universality (9) and (10), one gets a condition $m_0^2 > O(1)M_{1/2}^2$. The UFB conditions were also investigated for more general classes of models.

Possible Solutions:

(A) The CCB conditons of the type (16) and the UFB conditons on the parameters are inequalities that are not necessarily unnnatural and can be used as restrictions on the parameter space that are sufficient to free the model of possible cosmological constraints.

(B) These constraints can be completely relaxed by adding the R-parity couplings (13) with couplings which are below their upper limits just discussed – though not very far – while (14) are not needed so that there is no problem with proton decay. In a sense, the filling in of the UFB vaccua is a fair justification for R-parity explicit violation!

3.3 Supersymmetric FCNC problems

Problems:

The GIM mechanism explains why flavour is almost conserved in transitions with $\Delta Q_{\rm em} = 0$, and the SM predictions for these rare transitions agree with measurements within the experimental and theoretical uncertainties. Therefore FCNC has always been a major obstacle to theories that introduce flavour changing interactions. Examples are the multi-Higgs models or generic R-parity vioitg couplings. In the MSSM, the interactions are supersymmetric extensions of the SM ones but, as already stressed, the misalignment between sfermion and fermion mass matrices, due to (8), introduce family mixing in the couplings of gauginos and Higgsinos, spoiling the GIM mechanism for gauge couplings and the Glashow-Weinberg mechanism for Higgs couplings.

Let us consider for instance, the mass mixing in the $K^0 - \bar{K}^0$ system, a monument of particle physics. In the SM it is due to the CKM mixing in the couplings of the $W_\pm$ that are exchanged twice in the box diagram, resulting in a $s \longrightarrow d$ transition, but with a GIM suppression, a factor $m_c^2 \sin\theta_C / M_W^2$ in the case of two families.

In the MSSM, one can consider analogous diagrams with *both neutral and charged* gauginos (higgsinos are less important for the first two families) and the appropriate squarks (or sleptons) as the virtual states propagating in the loop diagrams. Let us consider the box with gluinos and squarks. Since the external quarks are s and d, we choose to work in a family basis where m_d is diagonal.The $\tilde{m}_Q^2$, $\tilde{m}_D^2$, A_D matrices in the down-squark mass matrix are not diagonal in this basis. The diagonalization introduces a unitary matrix in the gluino couplings, actually a 6x6 matrix since the L and R scalars mix, though the $L-R$ mixing is small as already explained. The necessarily large suppression factor will require for a strict relation among the mixings and the differences between mass eigenvalues. Actually, it is much easier to rephrase the issue in terms of the so-called *insertion approximation*, where the squark propagators, as well as the gaugino-higgsino propagators in the relevant case, are developped around the basis where quark masses are diagonal. Since the effects must be suppressed, the approximation is good (as far as one does not neglect possible differences in the diagonal entries of the mass matrices!). This is a development in the off-diagonal entries $(\tilde{m}_Q^2)_{ij}$, *etc* ... It is then is usual to define:

$$
\begin{aligned}
(\delta_L^D)_i j &= \frac{2(\tilde{m}_Q^2)_{ij}}{((\tilde{m}_Q^2)_{ii} + (\tilde{m}_Q^2)_{jj})} \\
(\delta_R^D)_i j &= \frac{2(\tilde{m}_D^2)_{ij}}{((\tilde{m}_D^2)_{ii} + (\tilde{m}_D^2)_{jj})} \\
(\delta_{LR}^D)_i j &= \frac{2(A_D m_d)_{ij}}{((\tilde{m}_D^2)_{ii} + (\tilde{m}_D^2)_{jj})}
\end{aligned}
\tag{20}
$$

and analogous parameters for the U and E sectors. The comparison with the experimental data on the neutral $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$, $B_d^0 - \bar{B}_d^0$ systems give bounds on the δ's such that the strongest are on the combinations [12]:

$$
\begin{aligned}
\sqrt{(\delta_L^D)_{12}(\delta_R^D)_{12}} &< 2.10^{-3} \, \left(\frac{m_{\text{squark}}}{500\text{GeV}}\right), \\
\sqrt{(\delta_L^D)_{13}(\delta_R^D)_{13}} &< 2.10^{-2} \, \left(\frac{m_{\text{squark}}}{500\text{GeV}}\right), \\
\sqrt{(\delta_L^D)_{23}(\delta_R^D)_{23}} &< 2.10^{-2} \, \left(\frac{m_{\text{squark}}}{500\text{GeV}}\right).
\end{aligned}
\tag{21}
$$

while the limits on δ^D_{LR} are less interesting if one takes into account the quark mass factor. These are limits on the real part of the parameters. The limits on the imaginary parts are much more constrained because they produce CP violation. In the absence of a theory for the CP phases in the MSSM one could take allow for large phases, at least in the flavour changing sector, and then further reduce the bounds like (21). For instance, the first bound in (21) should be reduced by an order of magnitude to accomodate for a large phase.

These limits have been obtained by defining (20) in the basis where m_d is diagonal. To keep the same definition when up-quarks are involved, or more generally, to avoid particularizing one basis, one has also to consider the quantities like

$$\frac{\left(\tilde{m}^2_Q\right)_{ii} - \left(\tilde{m}^2_Q\right)_{jj}}{\left(\tilde{m}^2_Q\right)_{ii} + \left(\tilde{m}^2_Q\right)_{jj}} \times \theta_{ij} \tag{22}$$

where the mass splitting between scalars is multiplied by the appropriate CKM matrix element. Indeed, to reduce FCNC in both the up and down quark sector would require a good alignment of $\tilde{m}^2_Q$ to *both* m_d and m_u.

The limits in (21) are small numbers. According to the *naturalness principle* as formulated by t'Hooft, namely, that small numbers are natural only if the symmetry increases when they vanish. They must be protected by a symmetry, and generated through symmetry breaking. Therefore one of the main challenges in supersymmetric particle physics is to understand the limits like (21) on supersymmetric FCNC effects from a theoretical viewpoint.

Possible Solutions:

(A) **Degeneracy** – The easiest to way to make the commutators in (8) to vanish is to assume that the scalar masses are degenerate, *i.e.*, family independent at some scale. As already noticed, the corrections due the Yukawa coupling asymetry in the RGE running is consistent with the observed FCNC effects. This is obvious if one compares (12) and (21). The scalar masses would have an $U(3)$ symmetry that would contrast with the hierarchy in the quark masses. In principle this is possible if the susy breaking is family blind. In models based on global susy –at least those which are known – it is not possible to give large masses to all the scalars while keeping the fermions light by a direct coupling to the goldstino direction, the direction of susy breaking. Therefore one has to introduce a hidden sector which is coupled through quantum loops to the MSSM sector. In the so-called *gauge mediated* models [13] this is achieved through the gauge sector. Since gauge couplings are family independant, the resulting scalar masses will inherit this property, and FCNC are not a problem in these models.

A particular case is the universal one in which all the scalars in the theory are degenerate at some scale, presumably Λ_{GUT} or $M_{\rm Planck}$. This would correspond to susy breaking along a direction which is coupled universally to the chiral multiplets, at least those of the MSSM. In principle this is possible in *gravity mediated* models [14], that will be discussed in more detail below. The dilaton, a necessary element in string theories, is an example of this property. We shall also see later how the flavour dependence of the theory could bias the universality.

Finally, one could assume partial degeneracy, for instance, between the first two families of squarks to avoid large contributions to the $K^0 - \bar{K}^0$ system. It is motivated by the approach to the fermion mass hierachy problem where the relative lightness of the first two families is explained by a SU(2) symmetry (or equivalent) which is broken in a second step. The main susy breaking is SU(2) invariant, the corrections only appearing at the second step. Actually, the degeneracy solution to FCNC suppression points towards a non-abelian horizontal family symmetry. It is less clear whether this is also what is suggested by the fermion flavour puzzle [4].

(B) **Alignment** – Even if the scalar mass eigenstates are not degenerate, the commutators in (8) could vanish or be small enough to suppress the FCNC effects. Phenomenologically one assumes that all commutators in (8) are very small, but, for since the up- and down-quarks themselves are not aligned,

$$\begin{aligned} \left[\tilde{m}_Q^2, m_d^\dagger m_d\right] &\approx 0 \\ \left[\tilde{m}_Q^2, V^\dagger m_u^\dagger m_u V\right] &\approx 0 \end{aligned} \tag{23}$$

where V is the CKM matrix. This suppresses the FCNC in the $K^0 - \bar{K}^0$ system, but if the squarks are not degenerate one expects effects proportional to (22) in the $D^0 - \bar{D}^0$ system since the squarks are not aligned to the up-quarks. This contrasts with the more radical suppression of FCNC effects in the previous solution.

Models possessing alignment at least in the sector of the two first families have been discussed in the literature. They should be viewed more as an existence proof than as an attractive solution to the FCNC problem. Typically they postulate two or more flavour symmetries and some pattern in their breaking while the quantum numbers are ajusted so that the needed alignment follows together with fermion mass hierarchy [15].

(C) **Decoupling** – The most current solution to FCNC problems in models beyond the SM is to decouple the effects by pushing the scales of the model high enough. Could one put a lower limit on the susy scale by following this conservative school of thought? Of course one cannot increase the masses of all the MSSM scalars and gauginos. The sector which is responsible for the electroweak symmetry breaking has to remain at a scale consistent with the value of m_Z or else some fine tuning would be required. This includes of course the Higgs parameters, $m_{H_{1,2}}$, $B\mu$, μ, as well as the masses in the top sector, and the gaugino masses, specially the gluino one. The usual, more or less compelling naturalness arguments lead to a limit of $O(1\ \text{TeV})$ for this sector. Since the main aim is to avoid effects where the first two families are more relevant, it has been suggested to assume that the first two generation sfermions are much heavier than those belonging to the third family [16]. Indeed, the limits in (21) reflect the decoupling for large squark masses, which effectively cut off the loop integrals, though it is a bit less efficient as the gauginos remain lighter. Roughly, one can also conclude from (21) that the squark masses have to be quite large, raising the question: how large can they be without disturbing the electroweak symmetry breaking? The first two families contribute to the running of the parameter $m_{H_2}^2$ at two loops but if the masses are large they compensate the additional gauge coupling factor. Applying the same naturalness principle as above suggests that these masses cannot exceed 5 or 10 TeV. This is not sufficient by itself for the FCNC supprssion, but can be combined with the other mechanisms above to accomplish it.

At the end of these lectures I shall discuss models that try to combine all these ingredients in a cocktail that could reduce some FCNC effects without killing them all.

3.4 Supersymmetric CP violations

(N.B. – Since this subject is fully presented in the lectures by Y. Nir, it is not included in these notes.)

4 Susy Geometry and Soft Terms

The phenomenological constraints on the flavour pattern of soft terms are particularly strong. They have to be fulfilled in terms of the susy breaking mechanism and its transmission to the MSSM sector. Let us first review in some detail the formalism of broken local supersymmetry within a quite general approach since we do not have any compelling susy breaking model.

4.1 Kähler geometry

The kinetic terms of $N = 1$ supersymmetric theories are defined from the Kähler geommetry of the complex manifold of the scalar fields z^Ain the chiral supermultiplets Φ^A with components (z^A, ψ^A, F^A). The Kähler metrics is defined from the Kähler potential $K(z^A, \bar{z}^{\bar{A}})$ as:

$$\partial_A \partial_{\bar{A}} K = K_{A\bar{A}} \left(z^B, \bar{z}^{\bar{B}} \right) \tag{24}$$

Of course one can also define $K((\Phi^A, \bar{\Phi}^{\bar{A}})$ in terms of the superfields. The kinetic terms in the supersymmetric lagrangian is:

$$\mathcal{L}_{\rm kin} = K_{A\bar{A}} \left(\partial_\mu z^A \partial^\mu \bar{z}^{\bar{A}} + i \bar{\psi}^{\bar{A}} \partial\!\!\!/ \psi^A + F^A F^{\bar{A}} \right) \tag{25}$$

with a $\sigma-$model structure of the Kähler type. Indeed, the Kähler potential $K((z^A, \bar{z}_{\bar{A}})$ is defined by the symmetries of the manifold, including the gauge symmetries. The natural scale of the manifold is the sugra scale, $M_{\rm Planck}$, at the classical level, so that we choose it as the scale unit. Quantum corrections produce Kähler deformations and introduce new scales, such as the perturbative scale dependence and thresholds. Hence the metrics (24) is field dependent

($\sigma-$model) and scale dependent. It is related to the wave-function renormalization so that the chiral supermultiplets get renormalized at a given scale by the vielbeins:

$$K_{A\bar{A}} = \zeta^a_A \bar{\zeta}^a_{\bar{A}} \tag{26}$$

(we shall avoid the ambiguous notation "$\zeta = K^{1/2}$". The metric is invariant under the Kähler transformations,

$$K(z^A, \bar{z}_{\bar{A}}) \to K(z^A, \bar{z}_{\bar{A}}) + g(z^A) + \bar{g}(\bar{z}_{\bar{A}}) \tag{27}$$

where g is analytic. Correspondingly, an analytic function transforms as (indices omitted for simplicity):

$$\begin{aligned} f(z) &\to e^{-g(z)} f(z) \\ e^{K/2} f(z) &\to e^{-i\mathrm{Im}\, g(z)} e^{K/2} f(z) \end{aligned} \tag{28}$$

One defines the covariant derivative by:

$$\begin{aligned} \nabla_A &= \partial_A + K_A - \Gamma^C_{AB} \\ &\to e^{-g(z)} \nabla_A e^{g(z)} \\ \nabla_A f(z) &\to e^{-g(z)} \nabla_A f(z) \end{aligned} \tag{29}$$

with the connexion,

$$\Gamma^C_{AB} = K^{\bar{D}C} \partial_A K_{B\bar{D}} \tag{30}$$

and an additional term K_A related to the Kähler transformations.

The Riemann curvature tensor is

$$\begin{aligned} R_{A\bar{B}C\bar{D}} &= \partial_A \partial_{\bar{B}} K_{C\bar{D}} - \Gamma^F_{AC} K_{F\bar{E}} \Gamma^{\bar{E}}_{\bar{B}\bar{D}} \\ &= \partial_A \partial_{\bar{B}} K_{C\bar{D}} - K^{\bar{E}F} \partial_A K_{C\bar{E}} \partial_{\bar{B}} K_{F\bar{D}} \end{aligned} \tag{31}$$

In some special situations the metrics can be approximately diagonal for some set of fields

$$K_{B\bar{C}} = \delta^C_B Z_B \left(z^A, \bar{z}_{\bar{A}}\right) \tag{32}$$

in which case one gets,

$$\begin{aligned} \Gamma^C_{AB} &= \partial_A \left(\ln \mathcal{Z}_B\right) \delta^C_B \\ R_{A\bar{B}C\bar{D}} &= \delta^D_C \mathcal{Z}_C \partial_A \partial_{\bar{B}} \left(\ln \mathcal{Z}_C\right) \end{aligned} \tag{33}$$

4.2 Superpotential

The superpotential is a chiral supermultiplet $W(\Phi^A)$ which is a function of the chiral fields Φ^A and, by extension, a holomorphic function $W(z^A)$ of the complex scalars z^A. It is invariant under the gauge and flavour symmetries,

$\delta W = \partial\ W/\partial\ \Phi_A \delta\Phi^A = 0$ (but has $R-$charge=2, when defined). The superpotential is protected by its holomorphy against perturbative quantum corrections. The parameters in the superpotential – Yukawa couplings and supersymmetric masses – are only renormalized by the wave-function renormalization, namely by the quantum deformations of the Kähler metric.

In the presence of supersymmetry breaking, the gravitino mass is given by the *vev* of the following expression,

$$m_{3/2} = e^{K/2} W(z^A), \tag{34}$$

which changes by a phase under Kähler tranformations.

4.3 Auxiliary fields

The auxiliary fields F^A and D^α carry the supersymmetry breaking effects. After the use of the field equations they take the expressions (we only display the bosonic dependence here, which is the one relevant in the following):

$$F_A = e^{K/2} \nabla_A W(z) = K_{A\bar{B}} \bar{F}^{\bar{B}} \tag{35}$$

for the auxiliary field of the chiral multiplet Φ_A, which gets a phase under the Kähler transformations, and

$$D^\alpha = i g_\alpha K_A \left(\delta_\alpha z^A\right) \tag{36}$$

for the auxiliary field in the vector multiplet, which depends on the action of the gauge symmetry on the chiral multiplets. These usually transform in linear representations,

$$\delta_\alpha \Phi^A = -i (T^\alpha)^A_B \Phi^B \tag{37}$$

The invariance of D^α under a Kähler transformation follows from the invariance of the analytic function $g(z)$ under the (complexified) gauge invariance,

$$\delta g(z) = \partial_A \delta z^A = 0 \tag{38}$$

For the so- called canonical metric, $K = \delta_{A\bar{B}} \bar{z}^{\bar{B}} z^A$, so that $K_A = \delta_{A\bar{B}} \bar{z}^{\bar{B}}$ in (36). In this case, with (37) one obtains the usual expression,

$$D^\alpha = g_\alpha \bar{z}_{\bar{A}} (T^\alpha)^A_B z^B \tag{39}$$

The auxiliary fields F^A couple to the generalized Yukawa couplings and supersymmetric mass terms in the superpotential as shown in (35). Notice that they are holomorphic only in the limit $M_{\mathrm{Planck}} \to \infty$ when all the gravitational couplings are neglected. In the simple case of a canonical metric,

$$F_A = e^{K/2} \frac{\partial W}{\partial z^A} + m_{3/2} \bar{z}_{\bar{A}} \tag{40}$$

4.4 Scalar potential and susy breaking

After the auxiliary fields are replaced by their expressions in (35) and (36), the scalar potential of a locally supersymmetric theory takes the form:

$$V = F^A F_A - 3|e^{K/2}W|^2 + \frac{1}{2}D^\alpha D_\alpha \tag{41}$$

The negative term, coming from the elimination of a sugra auxiliary field, is $-3m_{3/2}^2$ according to (34).

Supersymmetry breaking occurs when some of the auxiliary fields get *vev*'s: $\langle F^A \rangle \neq 0$ and/or $\langle D^\alpha \rangle \neq 0$. They give a positive contribution to the minimum of the scalar potential. In the limit global susy, namely, when $M_{\text{Planck}} \to \infty$ and $m_{3/2} \to 0$, these are the only contribution to the cosmological constant Λ_{cosm}. But in the context of sugra, they can be cancelled by the negative term in (41) in a theory with susy breaking and vanishing Λ_{cosm}. Of course, this is a major property of sugra theories, even if we are not yet able to give a satisfactory solution to the Λ_{cosm} puzzle. From now on we shall assume $\Lambda_{cosm} = 0$ so that $\langle V \rangle = 0$. Then,

$$m_{\frac{3}{2}} = \frac{1}{M_{PL}} \frac{\left\langle F^A F_A + \frac{1}{2} D^\alpha D^\alpha \right\rangle^{1/2}}{\sqrt{3}} = \frac{\left|\left\langle e^{K/2} W \right\rangle\right|}{M_{PL}^2} \tag{42}$$

This expression for the gravitino mass is just the counterpart of the *mass = coupling* $\times$ *v.e.v.* of the gauge particle associated to the broken local symmetry, which is the gravitino for local susy or sugra. Indeed, the gravitino mass is proportional to the overall amount of susy breaking which couples only through the sugra coupling, M_{Planck}^{-1}, to the gravitino. As we shall discuss later, the scalars also have a gravitational coupling to the $\langle F^A \rangle$ that leads to scalar masses of $O(m_{\frac{3}{2}})$. But the coupling through the goldstino, the helicity $1/2$ component of the gravitino, can be larger in principle. In this case, the gravitational coupling of susy breaking could be negligible as well as all typical sugra terms, and the theory would possess aproximate global susy.

Therefore one has to distinguish two cases of low energy effective theoris with broken local susy (remember that susy must be a local symmetry as it encompasses the local Poincaré invariance of gravitation):

(i)those where susy breaking effects are of $O(m_{\frac{3}{2}})$, and susy breaking couples mostly through sugra (*gravity mediation*), formally obtained in the limit $M_{\text{Planck}} \to \infty$ with $m_{3/2}$ fixed [14] ;

(ii)those where low energy susy breaking effects are much larger than $m_{\frac{3}{2}}$, obtained in the limit $M_{\text{Planck}} \to \infty$ with $m_{\frac{3}{2}} \to 0$, which can be discussed as approximated global susy theories; in this case the coupling to the matter supermultiplets that yield their scalar masses have to be quantum effects and it has been known since the beginning of supersymmetric particle physics that the best candidate is what is called *gauge mediation* [13] (more on that later).

4.5 Origin of soft terms

Let us now discuss in more detail the effects of the breaking of the local susy (and the effective global susy when applicable) at energies well below M_{Planck}. We neglect gravitational couplings, but keep those of $O(m_{3/2})$, which can be further neglected if this is justified. Actually, we want to stress the importance of the additional terms – with respect to the global susy limit – and also to show that they are closely related to the Kähler geommetry.

In a theory with broken gauge invariance, the effective lagrangian after the symmetry breaking displays new interactions obtained by shifting the relevant fields by the amounts of their *vev*'s. These new terms are called *soft* in the sense that they do not worsen the renormalization properties of the theory. For instance, in the Standard Electroweak Theory, the Higgs field H gets a *vev* that produces a series of soft terms which give masses to the various particles without spoiling the renormalizability:

$$\begin{aligned} \mathcal{L}^{\text{soft}}_{\text{SM}} &= \left(\underline{\langle H\rangle}\frac{\partial}{\partial H}+\frac{1}{2}\underline{\langle H\rangle}^2\frac{\partial_2}{\partial H^2}\right)\mathcal{L}_{SM} && (43)\\ &= \lambda_t\underline{\langle H\rangle}\bar{t}t+\ldots+\frac{1}{2}g_2^2\left|\underline{\langle H\rangle}\right|^2 W^{+\mu}W^-_\mu && (44)\end{aligned}$$

In a theory with global susy, the soft terms are obtained by a similar expansion in the auxiliary field *vev*'s $\langle F^A\rangle$ and $\langle D^\alpha\rangle$. The expansion stops at the quadratic terms (linear for the superpotential) because of the development in powers of the Grassmann variables $\theta,\bar\theta$. Therefore the soft terms are:

$$\mathcal{L}^{\text{soft}}_{\text{global}} = \underline{\langle F^A\rangle}\frac{\partial W}{\partial z^A}+h.c.+\underline{\langle F^A\rangle\left\langle \bar{F}^{\bar{A}}\right\rangle}K_{A\bar{A}}+\ldots+\underline{\langle D^\alpha\rangle}D^\alpha\left(z,\bar{z}\right) \quad (45)$$

If the supersymmetric theory was renormalizable in the first place, the addition of the soft terms preserve the renormalizability. Non-renormalizable (*e.g.* effective) Kähler metrics are needed to obtain a scalar mass through the second term in the r.h.s. of (45). Indeed the first term in (45)can only give a mass splitting between scalars and the corresponding fermions, a well-known feature that is no good in the case of matter supermultiplets. It also introduce analytic scalar interactions, the so-called A-terms. The last one, gives a D-type mass to the scalars, which cannot be the whole story since the property that the generator T^α associated to D^α is traceless by the anomaly condition implies that these terms are both positive and negative. (We shall discuss later the possibility of an "anomalous" $U(1)$.)

In a sugra theory, the soft terms are just a bit more involved [14, 17]:

$$\begin{aligned} \mathcal{L}^{\text{soft}}_{\text{local}} &= \left\langle F^A\right\rangle e^{K/2}\nabla_A W+h.c.+\left\langle D^\alpha\right\rangle D^\alpha\left(z,\bar{z}\right)\\ &+\left\langle F^A\right\rangle\left\langle \bar{F}^{\bar{B}}\right\rangle\left(\frac{1}{3}K_{A\bar{B}}K_{C\bar{D}}-R_{A\bar{B}C\bar{D}}\right)\bar{z}^{\bar{D}}z^C && (46)\end{aligned}$$

where the appropriate definition of the $D^{\alpha}(z,\bar{z})$ is given in (36).

The main differences with respect to (45) are in the geometric nature of the couplings of the $\langle F^A\rangle$ to the scalars. This "geometric transmission" is specially clear for the second term which contains two terms: one proportional to the total amount of F-type breaking, the same for all scalars (*i.e.*is universal, or flavour independent), and another with the Riemann curvature that depends on how the susy breaking directions are related to the different scalars by the Kähler geometry.

But, as already noticed, there is also a difference in the generalized A-terms, the first term in (46), because it is not an analytic scalar interaction.

4.6 Soft terms as remnants of hidden susy breaking

In order to get a better insight into the physical content of the soft terms (46), let us consider a situation quite close to what one expects from a sugra theory as the beyond-the-standard-theory candidate. We separate the chiral superfields Φ^A into:

(i) a *hidden sector*, z^a, which is related to the physics that one expects to become relevant at very high energies, to provide solutions to problems like susy breaking, flavour problems and grandunification;

(ii) a *MSSM sector*, Φ^i, holding the quarks, leptons, and Higgses, together with their susy partners (of course, MSSM stands here for any "low energy" theory).

If one considers the effective theory at energies far below M_{Planck}, the hidden fields have been integrated out, and only appear as classical variables. For the standard fields, we only retain the terms in the lagrangian which are of dimension less or equal to four, to obtain an effective renormalizable theory. Expanding the Kähler potential and keeping the relevant terms yields the effective one:

$$K = K_o\left(z^a,\bar{z}^{\bar{a}},\Lambda\right) + K_{i\bar{j}}\left(z^a\bar{z}^{\bar{a}},\Lambda\right)\phi^i\bar{\phi}^{\bar{j}}\ldots \tag{47}$$

where the metric for the MSSM fields depends on the hidden fields and on the scales Λ that define the effective theory. A similar expansion is done for the superpotential

$$W = W_o\left(z^a\right) + \mu_{ij}\left(z^a\right)\phi^i\phi^j + Y_{ijk}\left(z^a\right)\phi^i\phi^j\phi^k + \ldots \tag{48}$$

Then assume that susy is broken through some auxiliary fields in the chiral hidden sector, F^a and auxiliary fields in the hidden gauge sector (if any) D^{α}. The soft terms are obtained by applying the previous formulae. However, since we want to work directly with the canonical metrics, we renormalize the fields by their vielbein:

$$\hat{\phi}^i = \zeta^i_k\phi^k \qquad \hat{\bar{\phi}}^{\bar{i}} = \bar{\zeta}^{\bar{i}}_{\bar{k}}\phi^{\bar{k}} \qquad \zeta^i_k\bar{\zeta}^{\bar{i}}_{\bar{l}} = K_{k\bar{l}} \tag{49}$$

From the quadratic term in (46) we obtain with this redefiniton to the physical, flat metrics, the following expression for the scalar masses [17, 18]:

$$\tilde{m}^2_{i\bar{j}} = m^2_{3/2}\delta_{i\bar{j}} - \hat{R}_{a\bar{b}i\bar{j}}\left\langle F^a\bar{F}^{\bar{b}}\right\rangle \tag{50}$$

where,

$$\hat{R}_{a\bar{b}ij} = R_{a\bar{b}k\bar{l}}(\bar{\zeta}^{-1})^{\bar{l}}_{i}(\zeta^{-1})^{k}_{j}$$
$$m^2_{3/2} = \frac{1}{3}e^{-K_o}|W|^2 = K_{a\bar{b}}\bar{F}^{\bar{b}}F^a \tag{51}$$

The scalar masses get two contributions: an *universal* one whose origin is clearly in supergravity, corrected by *Kähler curvature* terms which may carry some 'flavour' dependence. This shows the geometric interpretation of the flavour pattern of scalar masses as reflecting the flavour dependence of the Kähler metric. An interesting case are the so-called 'no-scale' models, where the curvature terms exactly cancel the term $m^2_{3/2}$ for all matter fields at the classical level.

From (46), concentrating on the cubic terms in the ϕ^i scalars, one gets, *before* field renormalization, the coefficient of the trilinear soft term (analytic in the ϕ^k):

$$\begin{aligned} A_{ijk}Y_{ijk} &= \langle F^a\rangle\, e^{-K_o/2}(\partial_a Y_{ijk} + K_a Y_{ijk} \\ &\quad -\Gamma^{i'}_{ai}Y_{i'jk} + \Gamma^{j'}_{aj}Y_{ij'k} + \Gamma^{k'}_{ak}Y_{ijk'}) \end{aligned} \tag{52}$$

One can reabsorb the factor $e^{-K_o/2}$ in the $Y_{ij'k}$ to remain consistent with the definition of the Yukawa couplings between fermions. There are three contributions to A_{ijk}: one is univerasl and depends on ∂_K. The first term is related to the dependence of the Yukawa couplings on the fields that contribute to susy breaking. The last contributions have a more geometric nautre.

In the simplified case of a diagonal effective metrics, $K_{i\bar{j}} = \delta i\bar{j} Z_i(z^a\bar{z}^{\bar{a}}, \Lambda)$, these relations become:

$$\begin{aligned} \tilde{m}^2_i &= m^2_{3/2} - \left\langle F^a\bar{F}^{\bar{b}}\right\rangle \partial_a\partial_{\bar{b}} \ln Z_i, \\ A_{ijk} &= \langle F^a\rangle \left(\partial_a \ln Y_{ijk} + \frac{1}{2}K_a - \partial_a \ln Z_i Z_j Z_k\right) \end{aligned} \tag{53}$$

where the factor $e^{-K_o/2}$ was included in the $Y_{ij'k}$. If one takes, as an example, $i \to H_2$, $j \to Q^j$, $k \to U^k$, one clearly see from either (52) or (53) that the matrices A^U and the Yukawa coupling matrix Y^U do not commute in general.

5 Some Examples

5.1 Modular invariant Kähler geometry

The soft terms have been studied in models with different kinds of moduli and modular dependence as possible phenomenological description of the low energy string physics. Here we discuss the simplest case, as an example of modular invariance [?] as a possible source of flavour asymmetry.

Take a single modulus field, T, as the hidden sector, and the MSSM scalars, Φ^i. Under the modular group $SL(2,Z)$ their tranformations are defined to be:

$$T \to \frac{aT - ib}{icT + d}\ ; \qquad \Phi^i \to (icT+d)^{n_i}\,\Phi^i \qquad\qquad (ad - bc = 1) \tag{54}$$

The integers n_i that define the tranformations of the Φ^i are the *modular weights.* The following Kähler potential is modular invariant up to a Kähler tranformation,

$$K = 3\ln\left(T+\bar{T}\right) + \left(T+\bar{T}\right)^{n_i}\Phi^i\Phi^{\bar{i}} \tag{55}$$

It exhibits flavour dependence if the n_i are different. Assume that susy is broken with vanishing Λ_{cosm}. Define the contribution of F_T to the susy breaking by the angle Θ, such that:

$$\frac{1}{3}F^T F_T = m_{3/2}^2 \cos^2\Theta \tag{56}$$

Then apply the formula in the previous sections to obtain for the scalar masses,

$$\tilde{m}_i^2 = m_{3/2}^2\left(1 - 3n_i\cos^2\Theta\right) \tag{57}$$

and for the trilinear coupling coefficients,

$$A_{ijk} = \left(n_i + n_j + n_k\right)\sqrt{3}m_{3/2}\cos\Theta \tag{58}$$

This example nicely shows that the universality condition is not necessarily more attractive even from a theoretical point of view.

5.2 Gauge mediated supersymmetry breaking

The global susy limit is assumed to be valid in these models, which possess a hidden sector with a gauge singlet X together with chiral multiplets, the *messengers*, Q, $\tilde{Q}$ transforming in conjugated non-trivial representations of the SM gauge group. The superpotential includes a term $W_M = XQ\tilde{Q}$, such that if X takes a large *vev* , which we also assume, this gives a mass $\langle X\rangle$ to the messengers that decouple from the low energy theory. As a final assumption, the hidden sector is coupled to the MSSM chiral fields only by gauge interactions.

The MSSM matter has a Kähler potential which is taken to be flavour diagonal,

$$K_{mssm} = Z_i(X\bar{X}, \Lambda)\phi^i\bar{\phi}^{\bar{i}} \tag{59}$$

where the metric Z_i carries the wave function renormalization. It feels the X field because $\langle X\rangle$ gives the decoupling threshold of the messengers. Indeed, the dependence of Z_i in the running coupling constants takes the well known expression:

$$\frac{Z_i(\mu)}{Z_i(\mu')} = \left(\frac{g^2(\mu')}{g^2(\mu)}\right)^{\frac{2C_i}{b}} \tag{60}$$

where we have taken a simple group, since the result is easily generalized, C_i is the Casimir eigenvalue of the field ϕ^i, and b is the coeffficient of the β-function of g^2,

$$g^{-2}(\mu) = g^{-2}(\mu') + \frac{b}{16\pi^2}\ln\frac{\mu^2}{\mu'^2} \tag{61}$$

Since the messengers decouple at the scale $\sqrt{X\bar{X}}$, there is a decrease in the running (61) above this scale, $b \to (b - b_Q)$. Therefore starting with $Z_i(\Lambda)$ at some scale (*e.g.*, the unification scale of the fundamental theory) and going down to the the scale μ, the regimen changes at $\sqrt{X\bar{X}}$ and one has,

$$\frac{Z_i(\mu)}{Z_i(\Lambda)} = \left(\frac{g^2(\Lambda)}{g^2(\sqrt{X\bar{X}})}\right)^{\frac{2C_i}{b-b_Q}} \left(\frac{g^2(\sqrt{X\bar{X}})}{g^2(\mu)}\right)^{\frac{2C_i}{b}} \tag{62}$$

which shows the dependence on $\ln\sqrt{X\bar{X}}$ (notice that $g^2(\mu)$ also depends on $\ln\sqrt{X\bar{X}}$). Now by applying (53),

$$\tilde{m}_i^2 = m_{3/2}^2 - \frac{|F_X|^2}{|X|^2}\left(\frac{\partial^2 \ln Z_i(X\bar{X})}{\partial \ln X \partial \ln \bar{X}}\right) , \tag{63}$$

where $m_{3/2}^2$ can be neglected since it is assumed to be small with respect to $|F_X|^2/|X|^2$, it is straightforward to get the well-known result,

$$\tilde{m}_i^2(\sqrt{X\bar{X}}) = \frac{2C_i g^4}{(16\pi^2)^2} b_Q \frac{|F_X|^2}{|X|^2} \tag{64}$$

by considering the limit $\mu^2 \to X\bar{X}$, or the correctly renormalized value of (64) at lower scales [20]. It is important to note that (64) is a 2-loop result,but here it follows from the one-loop wave function renormalization. The method has been also used for higher loop results.

This example illustrates how the Kähler metric encodes the RGE evolution of the theory.

5.3 Decoupling of a flavour theory

If a flavour theory is introduced at some scale M_F to solve the fermion flavour problems, the sfermion masses can get a flavour dependence because the flavour symmetry breaking fields tend also to develop some induced supersymmetric breaking through their auxiliary F-components, and to transmit it to the sfermions of the theory. For illustration, consider a Froggatt-Nielsen type flavour model [21], by making the set of assumptions: (i) a U(1) symmetry with charge Q_i associated to each Φ_i; (ii) a field ϕ with charge $Q = -1$ with $\langle\phi\rangle \neq 0$ that breaks the flavour U(1) symmetry.

The trilinear Yukawa couplings, Y_{ijk} is forbidden unless $Q_i + Q_j + Q_k = 0$. But in an effective theory at the scale M_F (which could be M_{Planck} or lower) non renormalizable interactions with powers of ϕ/M_F are present or are generated by integrating out some heavy fields. Therefore the forbidden Yukawas are replaced by a field dependent one,

$$Y_{ijk} \propto \left(\frac{\phi}{M_F}\right)^{Q_i+Q_j+Q_k} \tag{65}$$

so compensating the U(1) charge imbalance. In this way, it is possible to generate hierachies in the Yukawa coupling and, therefore, of fermion masses and mixings, through the powers of a relatively small number, $\langle\phi\rangle/M_F$, in the effective Yukawa couplings.

The Kähler metric of the Φ_i will depend on ϕ at the fundamental level or as an effective metric. Let us assume that supersymmetry is broken, producing a gravitino mass $m_{3/2}^2$, but with family independent or universal susy breaking in the MSSM sector.

Defining ϕ such that its metric becomes canonical, one has for the corresponding auxilairy field,

$$\langle F_\phi \rangle = \left\langle \frac{\partial W}{\partial \phi} \right\rangle - m_{3/2}\langle\bar{\phi}\rangle \tag{66}$$

One expects, and we shall check it in an example, that $\langle F_\phi \rangle \sim O(m_{3/2}\langle\bar{\phi}\rangle)$, corresponding to an *induced susy breaking* along the ϕ direction [22, 23]. This can be avoided only by an *ad hoc* choice of the superpotential, not necessarily consistent with the flavour theory.

The effective superpotential and the effective metrics of the Φ_i depend on ϕ from the integration on heavy states and loop effects as well. This will be made more explicit in the example below, but (65) shows the dependence expected in the superpotential. By applying the formulae displayed in the previous section, one sees that the corresponding A-terms will have a contribution proportional to $m_{3/2}$ and to the same power of $\frac{\langle\phi\rangle}{M_F}$ as the corresponding Y_{ijk}. The ϕ dependence of the Kähler metric introduce a flavour dependence in the scalar masses, $\tilde{m}^2_{i\bar{j}}$. Moreover, if the U(1) is gauged, the scalars get D-type masses proportional to their charges (more below).

Therefore, one generically expects a non-trivial flavour structure of the soft terms of $O(m_{3/2})$ from the hidden sector associated to a flavour theory. In the attractive class of the models that now go by the name of 'gravity mediated', the susy breaking are of $O(m_{3/2})$ and the flavour theory effects are expected to affect any kind of universality of the primordial susy breaking. On the contrary, in the now called 'gauge mediated' models, where the susy breaking splitting between the MSSM fermios and sfermions is much larger than $O(m_{3/2})$ one expects to be safe if M_F is large enough to decouple the flavour theory before susy breaking.

Nonetheless, a rich flavour pattern in the soft terms would provide us with very constraining conditions on the flavour theory that could usefully complement the fermion mass hierarchy puzzle.

6 An Example with Anomalous U(1) and Inverse Hierarchy

6.1 The Froggatt-Nielsen paradigm

The smallness of the fermion mass ratios and mixing angles faces us with a problem of naturalness. The direction initiated by Froggatt and Nielsen [21] to understand such a hierarchical pattern goes as follows:

(i) The key assumption is a gauged horizontal $U(1)_X$ symmetry violated by the small quark masses so that small Yukawa couplings are protected (forbidden) by this symmetry. Gauging the symmetry avoids massless Goldstone bosons when the symmetry is broken. The effective $U(1)_X$ symmetric theory below some scale M_F is supposed to be natural to the extent that all parameters are of $O(1)$. The scale M_F is the limit of validity of the effective theory. The X-charges of quarks, leptons and Higgses are free parameters to be fixed *a posteriori* and simply denoted q_i,u_i, d_i,ℓ_i, e_i,h_1,h_2, for the different flavours, where $i = 1, 2, 3$ is the family index.

(ii) One (or more) Froggatt-Nielsen field ϕ, a SM gauge singlet is introduced, and the $U(1)_X$ normalized so that $X_\phi = -1$. The effective (non-renormalizable) $U(1)_X$ allowed couplings are of the form $g^U_{ij}(\phi/M_F)^{q_i+u_j+h_2}Q^iU^jH_2$, with analogous expressions for the H_1 couplings to down quarks and leptons. The coefficients g^U_{ij}, etc, are taken to be natural, *i.e.*, of $O(1)$, unless they are required to vanish by the $U(1)_X$ symmetry.

(iii) The small parameter λ is identified with the ratio $(< \phi > /M_F)$ as the $U(1)_X$ symmetry is broken by the *Phi vev*. Below the scale $< \phi >= \lambda M_F$, one recovers the SM with the effective Yukawa coupling matrices given by

$$Y^U_{ij} = \lambda^{|q_i+u_j+h_2|} \qquad Y^D_{ij} = \lambda^{|q_i+d_j+h_1|} \qquad Y^E_{ij} = \lambda^{|\ell_i+e_j+h_1|} \tag{67}$$

The Yukawa matrix entries corresponding to negative total charge should vanish but these zeroes are filled by the diagonalization of the λ -dependent metrics.

The X-charges are now chosen to fit the hierarchy in the mass eigenvalues and mixing angles. The experimental masses (at $O(M_{Planck})$) of the third families give: $h_2 + q_3 + u_3 = 0$ and $x = h_1 + q_3 + d_3 = h_1 + \ell_3 + e_3$, where the parameter $\lambda^x = m_b \tan\beta/m_t$. With this restriction the Yukawa couplings depend only on the charge differences $q_i - q_3$, $u_i - u_3$,..., $e_i - e_3$ and x.

An important question was first investigated in ref. [15] in the Froggatt-Nielsen framework. Just like the Yukawa couplings, the soft susy breaking terms contain powers of the ϕ -field to implement the $U(1)_X$ symmetry. The scalar mass matrices have a corresponding hierarchy among their elements, so that $(\tilde{m}^2)_{i\bar{j}} \propto \lambda^{|X_i - X_j|}$. Even in the flavour basis that diagonalizes quark mass matrices, the squark mass matrices will be non-diagonal. One solution [15] is to add another abelian symmetry and a smaller scale. In this case it is possible to strongly suppress $(\tilde{m}^2_D)_{12}$. More precisely, the charge pairs are chosen to align the squark masses to the down-quark ones. Interestingly enough, the model predicts large $(\tilde{m}_U^2)_{12}$ leading to sizeable $D\bar{D}$ mixing that could be experimentally tested, as already pointed out here above.

6.2 A few questions and possible answers

The attractive idea of Froggatt and Nielsen raises some relevant questions, we would like now to argue that they all find satisfactory answers in the supergravity framework:

(A) What fixes the scale M_F of the effective theory possessing the horizontal symmetry? The obvious solution is to identify M_F to the highest physical scale, the Planck mass, $M_{\rm Planck}$. A supergravity theory is non-renormalizable as it contains powers of the Newton constant, the inverse of $M_{\rm Planck}$, an is supposed to arise from a theory of quantum gravity.

(B) What gives rise to the "small number" λ? An appealing suggestion is to relate it to the coefficient that appears in the Green-Schwarz mechanism, as folows. In general the horizontal $U(1)_X$ potentially has (triangle) anomalies with the other gauge symmetries, including the SM gauge group. Models exist where all these anomalies vanish. However, the more promising models [24] rely upon the Green-Schwarz mechanism: the shift of a scalar, an axion, under the $U(1)_X$ gauge transformations can cancel all the anomalies if and only if their coefficients obey an appropriate proportionality factor. Now, it can be shown that this requires a Fayet-Iliopoulos term, with a coefficient $\xi^2 = M^2_{\rm Planck}{\rm tr}X/6(16\pi)^2$. The minimization of the scalar potential then fixes $\phi^2 = \xi^2$. With $M_F = M_{\rm Planck}$, this defines the number $\lambda = \sqrt{6trX}/48\pi$, which can be small enough for reasonable values of trX (see below).

(C)How can such flavour theories be experimentally tested? Indeed, the charges are 'fitted' to reproduce the masses and mixings (with some predicted relations) up to some coefficients of $O(1)$, and one needs some predictions that can be experimentally measured. If one applies the general supergravity fromalism, rather than general considerations of $U(1)_X$ invariance, one gets more strict constraints on the soft terms, which are related to the fermion mass hierarchy. A striking consequence is an *inverse hierarchy* pattern for the sfermion masses.

In the rest of these notes, we first discuss point (B) above and then apply the general results to a special model to illustrate (C).

6.3 The Green-Schwarz mechanism

The introduction of the $U(1)_X$ gauge symmetry besides the SM gauge symmetry brings us to the fundamental question of gauge anomalies. Indeed, by a gauge transformation $X_\mu \to X_\mu + \partial_\mu \varepsilon\,(x)$, the lagrangian transforms as:

$$\delta \int F^{A\mu\nu}F^A_{\mu\nu} = \frac{\mathcal{A}_A}{8\pi^2}\int \varepsilon \tilde{F}^{A\mu\nu}F^A_{\mu\nu} \tag{68}$$

where $\mathcal{A}_A = {\rm tr}XT_A^2$, with $A = 3, 2, 1, X$, associated to the generators of each factor in the overall gauge group $SU(3)\otimes SU(2)\otimes U(1)\otimes U(1)_X$, If the coefficients

$\mathcal{A}_A$ do not vanish, the theory have anomalies that spoil the gauge invariance. It was found in string theories [25] that a chiral multiplet with a dilaton-axion complex scalar S allows for a cancellation of these anomalies. The gauge term in the supersymmetric lagrangian becomes,

$$
\begin{aligned}
\int d^2\theta \frac{S}{4k_A} W^{A\alpha} W^A_\alpha + h.c. &= \frac{\mathrm{Re}S}{4k_A} F^A_{\mu\nu} F^{A\mu\nu} + i\frac{\mathrm{Im}S}{4k_A} \widetilde{F}^A_{\mu\nu} F^{A\mu\nu} \\
&+ \frac{F^S}{4S} \lambda^\alpha \lambda_\alpha + \mathrm{h.c.} \ldots
\end{aligned} \tag{69}
$$

so that the gauge couplings are given by the *vev* of the real part of S, the dilaton, and the Kac-Moody levels k_A, namely,

$$
k_A g_A^2 = g^2 = (\mathrm{Re}S)^{-1} \ , \quad \forall A = 3, 2, 1, X. \tag{70}
$$

The other terms in (69) express the gaugino masses in terms of the *vev*'s of F_S.

If the anomalies are in a well defined relation,

$$
\frac{\mathcal{A}_A}{k_A} = \frac{\mathrm{tr}X}{24} \quad , \quad \forall A = 3, 2, 1, X. \tag{71}
$$

it is possible to compensate for the anomalies. The *r.h.s.* in (71) is the coefficient of the gravitational anomaly, which has also to vanish, as it does, *e.g.*, for the hypercharge, Y. All the anomalies can be compensated if S also transforms under $U(1)_X$, by a shift of its imaginary part, the axion,

$$
S \to S + \frac{i}{2}\delta_{GS}\varepsilon\,(x) \qquad \delta_{GS} = \frac{\mathrm{tr}X}{192\pi^2} \tag{72}
$$

This important property allows the abelian charge X to be conserved. In more recent investigations of string theories, it has been noticed that multi-dilaton theories are allowed, with the possibility of cancelling different anomalies with their different axion partners.

Since S is now transforming under $U(1)_X$, it will also appear in the corresponding $D-$term, according to (36), as is fixed by the supergravity theory. The Kähler potential for the dialton is $K(S, \bar{S}) = -\ln(S + \bar{S})$ leading to the final form of the D_X auxiliary field for the anomalous $U(1)_X$, with the Green-Schwarz mechanism:

$$
\begin{aligned}
iK_S\delta S &= \frac{\delta_{GS}}{2(S+\bar{S})} = \frac{g^2}{4}\delta_{GS} \equiv \frac{\xi^2}{M_{\mathrm{Planck}}} \\
D_X &= g_X\left(\min(X_i, X_j) K_i \Phi^j - K_\phi \phi + \xi^2\right) \\
&\to g_X\left(X_i\left|\Phi^i\right|^2 - |\phi|^2 + \xi^2\right)
\end{aligned} \tag{73}
$$

In (73) we have included (without further notations) the form obtained with the canonical metric, and we have put the M^2_{Planck} factor to explicitly account for the units. This result was first found by a string calculation [26].

The output theory has a Fayet-Iliopoulous term for the $U(1)_X$ that for reasonable values of tr$X$ is about one order of magnitude below M_{Planck}. This suggest to try to relate it to the small parameter that is needed in flavour theories, in particular in the Froggatt-Nielsen one, to explain the fermion mass hierarchy.

6.4 Inverse hierarchy model

Models with inverse hierarchies, in which the sfermions of the first two families are significantly heavier than those of the third one, have been discussed in several contexts. Models with an anomalous $U(1)_X$ naturally lead to this phenomenologically interesting situation. In particular, as already discussed, this could be useful to suppress FCNC effects.

The model presented here below is a very simplified one. It is a special case of a model that includes modular fields, allowing for a slightly more general spectrum [22]. Other approaches combine simple models of dynamical susy breaking [23].

Let us assume:

(i) The Froggatt Nielsen flavour theory, with an anomalous $U(1)_X$ as the flavour symmetry, as discussed above, with the charges fixed by a fit to the fermion masses and the anomaly condition (71). The scale $M_F = M_{\text{Planck}}$ and only one field ϕ is introduced, just like here above. There are several choices for the charges that are suitable, depending also on the agreement with the fermion spectrum that is required. We shall not be very concerned by this aspect of the problem here.

(ii) The dilaton multiplet S compensates for the anomalies and fixes the coupling constants. This is a very strong assumption since we do not know how to obtain a dilaton *vev* in these theories. This is a good reason to assume it without more discussion. Then the results just derived apply, including (73).

(iii) Susy is primarily broken only by the auxiliary field F_S of S, with $\Lambda_{cosm} = 0$, so that $\langle F^S/2S\rangle = \sqrt{3}m_{3/2}$. This is unrealistic, but the model is not realistic either! It yields a gaugino mass $M_{1/2} = \sqrt{3}m_{3/2}$.

(iv) The theory is natural so that the Yukawa couplings have the hierarchical structure (67). The Kähler potential of the matter fields has a dependence on ϕ dictated by the naturalness and the $U(1)_X$ invariance. In particular the non-diagonal entries in the metrics are of the form,

$$K_{i\bar{j}} = z_{i\bar{j}}\lambda^{|X_i - X_j|} \tag{74}$$

where the $z_{i\bar{j}}$ are numbers of O(1) in a natural theory. Of course, there is no term in the potential containing only ϕ.

The coupling of S to matter is taken to be only through supergravity in this simple approach. Therefore the susy breaking by F_S only produces a universal mass and a univesal A-term for the scalar fields Φ_i as well as for ϕ. Therefore, omitting the other fields, the scalar potential for ϕ is

$$V(\phi) = m_{3/2}^2 |\phi|^2 + \frac{1}{2} g_X^2 \left(\xi^2 - |\phi|^2\right)^2 \tag{75}$$

where the last term is the D-term with the Fayet-Iliopoulos term ξ. Since $\xi \gg m_{3/2}$, the minimization gives,

$$\xi^2 \approx |\phi|^2 \qquad \langle D_X \rangle \approx g_X^{-1} m_{3/2}^2 \tag{76}$$

The first equation gives the scale of the $U(1)_X$ breaking, which is ξ, and defines the 'small number' $\lambda = \xi / M_{\rm Planck} = \sqrt{{\rm tr} X}/16\sqrt{6}\pi$. The second one displays the induced D_X breaking. It can be checked that this is the only minimum by a detailed study of the flat directions with the general method already discussed. At the minimum of the potential,

$$F_\phi = m_{3/2}\phi = \lambda m_{3/2} M_{\rm Planck} \tag{77}$$

which is the induced susy breaking along the ϕ direction.

Finally, the computation of the soft terms yields relatively simple results (since this is a simplified model),

$$\begin{aligned} \tilde{m}_{i\bar{j}}^2 &= m_{3/2}^2 \left(1 + X_i \delta_{i\bar{j}} - \frac{1}{2} |X_i - X_j| \lambda^{|X_i - X_j|} z_{i\bar{j}}\right) \\ A_{ij}^U &= \left(-M_{1/2} + (h_2 + q_i + u_i) m_{3/2}\right) \delta_{ij} \\ &\quad - \left(\frac{1}{2}|q_i - q_j| \lambda^{|q_i - q_j|} z_{i\bar{j}}^Q + \frac{1}{2} |u_i - u_j| \lambda^{|u_i - u_j|} z_{i\bar{j}}^U\right) m_{3/2} \end{aligned} \tag{78}$$

together with similar results for A^D and A^E.

The first (universal) term in these equations correspond to the F^S susy breaking, with $M_{1/2} = \sqrt{3} m_{3/2}$, in the case of this model. The second term in $\tilde{m}_{i\bar{j}}^2$ is the contribution to the scalar masses from the D_X susy breaking, proportional to the $U(1)_X$ charges. The other contributions come from the induced F^ϕ susy breaking. It is worth noticing that most of this form is preserved in more general models.

For the third family one finds , from $h_2 + q_3 + u_3 = 0$ and $x = h_1 + q_3 + d_3 = h_1 + \ell_3 + e_3$,

$$A_t = -M_{1/2} \qquad A_b = A_\tau = -M_{1/2} + x m_{3/2} \tag{79}$$

for the diagonal A terms in the usual notation, and for the scalar (mass)^2 one obtains the relations,

$$\begin{aligned} \tilde{m}_{Q_3}^2 + \tilde{m}_{U_3}^2 + \tilde{m}_{H_2} &= M_{1/2}^2 \\ \tilde{m}_{Q_3}^2 + \tilde{m}_{D_3}^2 + \tilde{m}_{H_1} &= M_{1/2}^2 + x m_{3/2}^2 \\ \tilde{m}_{L_3}^2 + \tilde{m}_{E_3}^2 + \tilde{m}_{H_1} &= M_{1/2}^2 + x m_{3/2}^2 \end{aligned} \tag{80}$$

The most interesting results are for the differences between the diagonal terms,

$$\tilde{m}_i^2 - \tilde{m}_j^2 = (X_i - X_j)m_{3/2}^2 \qquad A_{ii} - A_{jj} = (X_i - X_j)m_{3/2} \tag{81}$$

Remarkably enough, these results remain more general, and are also found in models where the Kähler metric has flavour dependent modular invariances.

Therefore the spectrum is completely fixed by the charges. Notice that in many models of the Froggat-Nielsen type the following combinations are fixed:

$$\begin{aligned} \lambda^{q_i+u_i-q_j-u_j} &\approx \left(m_{u_i}/m_{u_j}\right) \\ \lambda^{q_i+d_i-q_j-d_j} &\approx \left(m_{d_i}/m_{d_j}\right) \\ \lambda^{\ell_i+e_i-\ell_j-e_j} &\approx \left(m_{e_i}/m_{e_j}\right) \end{aligned} \tag{82}$$

This displays the inverse hierarchy pattern for the scalars with respect to the fermions: light fermions are associated to large positive $U(1)_X$ charges, therefore their scalars are heavier, as expressed by (81), in units of $m_{3/2}^2$. Namely, with the value of the Cabibbo angle for λ, [22]

$$\frac{1}{2}\sum_{L,R}(\widetilde{m}_i^2 - \widetilde{m}_j^2) \approx \frac{1}{3}m_{3/2}^2 \ln{(m_i/m_j)}. \tag{83}$$

Finally, let us study the FCNC problems in the context of this model and its generalizations.With only one ϕ -field , the acceptable $U(1)_X$ charge assignments yield much too large FCNC effects in K-physics, see (21). However there is still enough freedom to choose, *e.g.*,$d_1 = d_2$ and $e_1 = e_2$, to suppress the most dangerous processes. Notice that this is not only due to the $U(1)_X$ symmetry: for matrix elements in a sector of degenerate charges the abelian symmetry gives no restriction, implying a generic non diagonal matrix by the naturalness principle. Instead, by a more detailed calculation one gets a diagonal degenerate matrix [22].

In the stringest case of Δm_K, the relevant factor is

$$(\delta_L^D)_{12}(\delta_R^D)_{12} \approx \frac{(d_1-d_2)(q_1-q_2)}{(d_1+d_2+\gamma)(q_1+q_2+\gamma)}\frac{m_d}{m_s} \tag{84}$$

where γ, relatively large in this models (≈ 20 in this simple model) accounts for the renormalization to low energy of the masses (the mass differences remain fixed). Now by putting $d_1 = d_2$, (84) vanishes, leaving only the contribution due to $(\delta_L^D)_{12} \approx 2m_d/(2+\gamma)m_s$, which is consistent with the present limits.

Therefore, the model, with this assignment avoids the FCNC problem by combining the three ingredients:

(a) alignment: the off diagonal matrix elements are suppressed by relatively high powers of λ, but they vanish for equal charges;

(b) degeneracy: degenerate charges lead to degenerate scalars, even if this can only be applied to a few charges;

(c) decoupling: the inverse hierarchy implies that the scalars associated to light fermions, which contribute mostly to the dangerous processes (Δm_K, e.d.m. of the neutron and the electron) are heavier, providing an additional suppression factor.

Of course, the whole approach has many arbitrarinesses and some choices are obviously *ad hoc* . But this discussion leads us to the conclusion that the sfermion spectrum could have a rich flavour pattern, give important hints on the flavour puzzles and still be consistent with FCNC – and CP violation – effects.

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